

CONSOLIDATION OF SOILS

A stress increase caused by the construction of foundations or other loads compresses soil layers. *The property of the soil due to which a decrease in volume occurs under compressive forces is known as **compressibility of soil**.* Compressibility is related to the magnitude of effective stress acting on the soil at that time. The compression is caused by

- (a) Deformation and relocation of soil particles
- (b) Compression and expulsion of air in the voids
- (c) Expulsion of water from voids

When the soil is fully saturated, compression of soil occurs mainly due to the expulsion of water. ***The compression of a saturated soil under a steady static pressure is known as consolidation.***

Compression of the soil below a structure causes settlement of the structure, which, is the vertical downward movement of the structure due to volume decrease of the soil on which it is built. In general, the total settlement in soil caused by loads may be divided into three broad categories:

1. **Immediate settlement (or elastic settlement)**

Immediate settlement (S_i or ρ_i) is caused by the elastic deformation of dry soil and of moist and saturated soils without any change in the moisture content. Immediate settlement calculations are generally based on equations derived from the theory of elasticity.

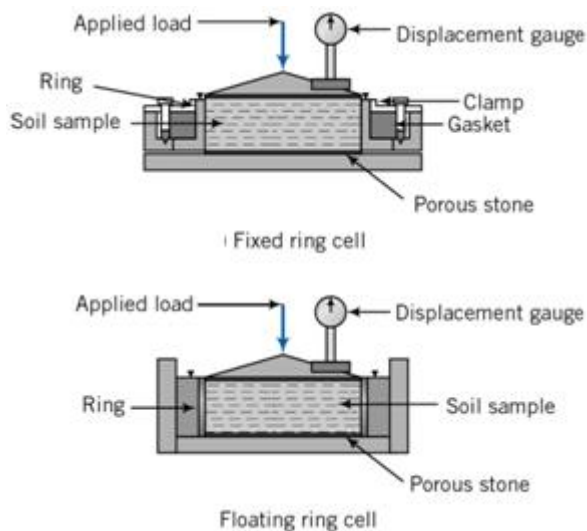
2. **Primary Consolidation Settlement**

Primary consolidation settlement (S_{pc} or ρ_{pc}) is the result of a volume change in saturated cohesive soils because of expulsion of the water that occupies the void spaces.

3. **Secondary Consolidation Settlement**

Secondary consolidation settlement (S_c) is observed in saturated cohesive soils and is the result of the plastic adjustment of soil fabrics. It is an additional form of compression that occurs at constant effective stress.

Consolidation test



Oedometer test

A disc of soil is enclosed in a stiff metal ring and placed between two porous stones in a cylindrical container filled with water. A metal load platen mounted on top of the upper porous stone transmits the applied vertical stress (vertical total stress) to the soil sample. Both the metal platen and the upper porous stone can move vertically inside the ring as the soil settles under the applied vertical stress. The ring containing the soil sample can be fixed to the container by a collar (fixed ring cell) or can move freely in the vertical direction (floating ring cell).

The fixed ring cell can also be used as variable head permeability test apparatus. For this purpose, a piezometer is attached to the base of the cell.

Specimens with diameters ranging from 60 mm to 100 mm and thickness ranging from 15

mm to 30 mm are used. The ratio of diameter to height is between 2.5 and 5. This ratio is usually taken as 3. Specimens with large diameter to height ratios suffer from disturbance to soil structure due to trimming, while those with smaller ratios have greater side friction.

An initial seating load of 5 kN/m² (2.5 kN/m² for very soft soils) is applied to prevent swelling. The load is applied till there is no change in the dial guage reading or 24 hours whichever is less.

Incremental loads, including unloading sequences, are applied to the platen. A dial gauge measures the settlement of the soil at various fixed times, under each load increment. Each loading increment is allowed to remain on the soil until the change in settlement is negligible and the excess pore water pressure developed under the current load increment has dissipated. For many soils, this usually occurs within 24 hours, but longer monitoring times may be required for exceptional soil types, e.g. montmorillonite.

It is usual practice to double the previous load in each increment. The successive pressures usually applied are 20, 40, 80, 160, 320 and 640 kN/m². For each load increment or decrement, the dial guage readings are noted at 0.25, 1.0, 2.25, 4.0, 6.25, 9.0, 12.25, 16.0, 20.25, 25, 36, 49, 64, 81, 100, 12, 144, 169, 196, 225, 289, 324, 400, 500, 600 and 1440 minutes. After consolidation under the final load increment is complete, the loads are reduced to one-fourth of the previous load and dial guage readings are noted for each load decrement till the load is reduced to initial seating load.

The data obtained from the one-dimensional consolidation test are as follows:

- Initial height of the soil, H_o , which is fixed by the height of the ring.
- The current height of the soil at various time intervals under each load (time-settlement data).
- The water content at the beginning and at the end of the test, and the dry weight of the soil at the end of the test.

Determination of void ratio at various load increments

The results of the consolidation test are plotted in the form of a plot between the void ration and the effective stress. It is therefore required to determine the void ratio at various load increments. There are two methods:

1. Height of solids method – Applicable to both saturated and unsaturated soils
2. Change in void ratio method – Applicable only to saturated soils.

1. Height of solids method

The equivalent height of solids is determined from the dry mass of the soil. The height of solids is given by

$$H_e = \frac{V_s}{A} = \left(\frac{M_s}{G\rho_w} \right) \cdot \frac{1}{A}$$

Where H_e = height of solids, V_s = volume of solids, M_s = dry mass of solids, G = specific gravity of solids, A = cross sectional area of specimen.

From the definition of voids ratio

$$e = \frac{V - V_s}{V_s}$$

$$e = \frac{(AxH) - (AxH_s)}{(AxH_s)} = \frac{H - H_s}{H_s}$$

Where H is the total height (total thickness).

The total thickness is measured once either at the beginning or at the end of the test. At other stages of loading, the thickness H is worked out from the measured thickness and the difference in dial gauge readings.

2. Change in void ratio method

The final void ratio e_l corresponding to the complete swelling conditions after the load has been removed, is determined from its water content, using the equation

$$e_l = wG$$

From the definition of void ratio

$$e = \frac{V - V_s}{V_s} = \frac{V}{V_s} - 1$$

$$V = V_s(1 + e)$$

$$A x H = V_s(1 + e) \dots\dots\dots(a)$$

By partial differentiation,

$$A dH = V_s \cdot de \dots\dots\dots(b)$$

From (a) and (b)

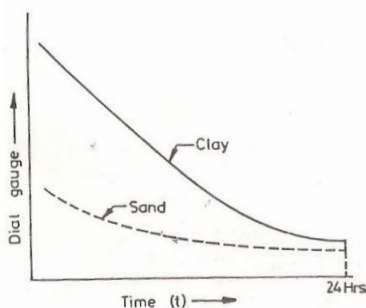
$$\frac{dH}{H} = \frac{de}{1 + e}$$

$$\Delta e = \frac{(1 + e)\Delta H}{H}$$

ΔH is the change in thickness at the end of the test.

Consolidation test results

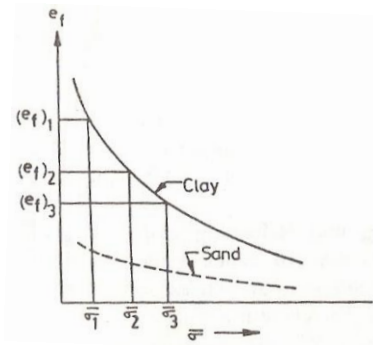
(1) Dial gauge reading-time plot



This plot is useful for the determination of coefficient of consolidation, which is useful for the determination of rate of consolidation in the field. This plot is drawn for each increment of load. The figure below shows a typical plot of Dial gauge reading Vs time for sand and clay. The thickness just after the application of the load increment ($t = t_0$) is maximum which decreases as the time increases. The decrease is rapid initially but decreases as time passes. There is practically no change in thickness after 24 hours. For sand, change

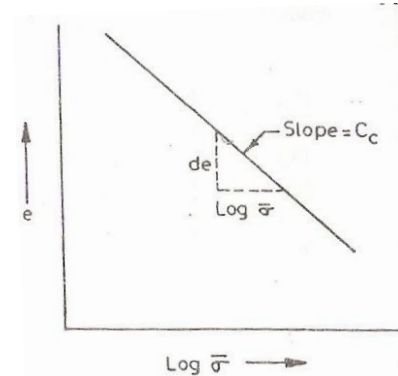
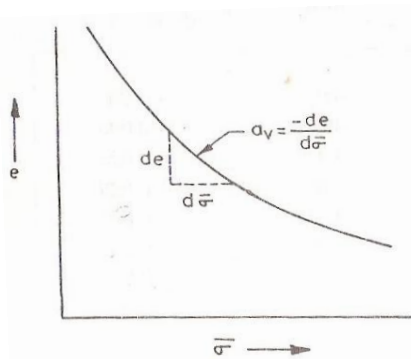
in thickness occurs very rapidly and stops after a few minutes due to high permeability of sand.

(2) Final void ratio-effective stress plot



This plot is required for determination of the magnitude of the consolidation settlement in the field. The thickness of the specimen after 24 hours of application of the load increment is taken as the final thickness for that increment. The final void ratio corresponding to the final thickness for each increment is determined using the height of solids or change in void ratio method.

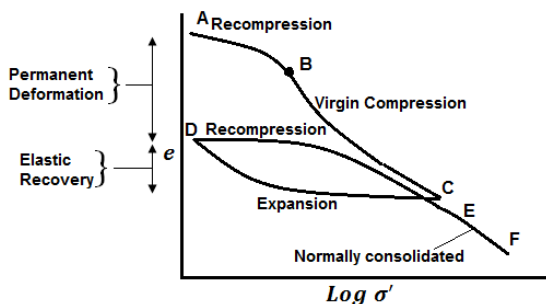
(3) Final void ratio – log σ' plot



The plot of e Vs $\log \sigma'$ is practically a straight line for a normally consolidated clay within the range of pressure usually encountered in practice.

(4) Unloading and reloading plot

The changes in void ratio or settlement are not linearly related to vertical effective stress. Typical plots by which soil settlement versus vertical effective stress is presented are shown below. The amount of settlement depends on the history of loading the soil and the vertical effective stress to be applied. The history of loading is described by the over-consolidation ratio - the ratio of the past vertical effective stress to the current vertical effective stress.



1. During the initial stages for pressures less than the past effective stress, the soil sample follows recompression curve, AB.

2. At pressures greater than the past effective stress, the sample follows virgin compression curve, BC, also known as normal consolidation line

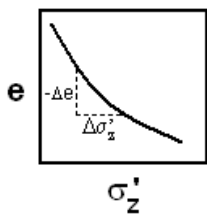
3. From 'C' when the sample is unloaded, sample expands and traces path CD (expansion curve - unloading)

4. Sample undergoes permanent strain due to irreversible soil structure and there is a small elastic recovery.
5. The deformation recovered is due to elastic rebound
6. When the sample is reloaded-reloading curve lies above the rebound curve and makes an hysteresis loop between expansion and reloading curves.
7. The reloaded soils show less compression.
8. Loading beyond 'C' makes the curve to merge smoothly into portion EF as if the soil is not unloaded.

Primary Consolidation parameters

Coefficient of compressibility, a_v

The coefficient of compressibility, a_v , is defined as decrease in void ratio per unit increase in effective stress. It is equal to the slope of the σ'_z vs e_z curve at the point under consideration.



$$a_v = -\frac{(e)_2 - (e)_1}{(\sigma'_z)_2 - (\sigma'_z)_1} = \frac{|\Delta e|}{(\sigma'_z)_2 - (\sigma'_z)_1} \quad \left(\frac{\text{m}^2}{\text{kN}} \right)$$

Coefficient of volume change or coefficient of volume compressibility or Modulus of volume compressibility, m_v

The slope BC in fig. 1c is called the *modulus of volume compressibility, m_v* , and is obtained from the plot of σ'_z vs ε_z as

$$m_v = -\frac{(\varepsilon_z)_2 - (\varepsilon_z)_1}{(\sigma'_z)_2 - (\sigma'_z)_1} = \frac{|\Delta \varepsilon_z|}{(\sigma'_z)_2 - (\sigma'_z)_1} \quad \left(\frac{\text{m}^2}{\text{kN}} \right)$$

where the subscripts 1 and 2 denote two arbitrarily selected points on the NCL. For most clays $m_v = 1 \times 10^{-3}$ to $1 \times 10^{-4} \text{ m}^2/\text{kN}$.

m_v may also be expressed as volumetric strain per unit increase in effective stress (this is inverse of bulk modulus). Thus,

$$m_v = \frac{-\Delta V/V_o}{\Delta \sigma'_z}$$

where V_o is the initial volume, ΔV is the change in volume and $\Delta \sigma'_z$ is the change in effective stress. The volumetric strain ($\Delta V/V_o$) can be expressed in terms of either void ratio or the thickness of the specimen.

- a. Let e_o be the initial void ratio and the volume of solids be unity. Therefore, the initial volume V_o is equal to $(1 + e_o)$. If Δe is the change in voids ratio due to change in volume ΔV , then $\Delta V = \Delta e$. Thus,

$$\frac{\Delta V}{V_o} = \frac{\Delta e}{1 + e_o}$$

Therefore,

$$m_v = \frac{-\Delta e / (1 + e_o)}{\Delta \sigma'_z}$$

- b. As the area of cross-section of the sample in the consolidometer remains constant, the change in volume is also proportional to the change in height. Thus $\Delta V = \Delta H$.

Therefore,

$$\frac{\Delta V}{V_o} = \frac{\Delta H}{H_o}$$

where H_o is the initial height. Therefore, m_v becomes,

$$m_v = \frac{-\Delta H / H_o}{\Delta \sigma'_z}$$

$$\Delta H = -m_v H_o \Delta \sigma'_z$$

The relationship between a_v and m_v can now be written as

$$m_v = \frac{a_v}{1 + e_o}$$

The coefficient of volume change depends upon the effective stress at which it is determined. Its value decreases with an increase in the effective stress.

Coefficient of compression or compression index, C_c

The *coefficient of compression or compression index, C_c* , is obtained from the plot of $\log \sigma'_z$ vs e as

$$C_c = \frac{e_2 - e_1}{\log \frac{(\sigma'_z)_2}{(\sigma'_z)_1}} = \frac{|\Delta e|}{\log \frac{(\sigma'_z)_2}{(\sigma'_z)_1}} \quad (\text{no units})$$

where the subscripts 1 and 2 denote two arbitrarily selected points on the NCL.

The compression index of a clay is related to its index properties, especially the liquid limit. Terzaghi and Peck gave the following empirical relationship for clays of low to medium sensitivity:

(a) for undisturbed soils: $C_c = 0.009(w_L - 10)$

(b) for remoulded soils: $C_c = 0.007(w_L - 10)$

where w_L = liquid limit (%). The value of C_c normally varies between 0.1 to 0.8.

Expansion index, C_e

The expansion index or swelling index (C_e) is the slope of the e Vs $\log \sigma'_z$ plot obtained during unloading.

$$C_e = \frac{\Delta e}{\log \frac{(\sigma'_z)_2}{(\sigma'_z)_1}}$$

Expansion index is much smaller than compression index.

Recompression index, C_r

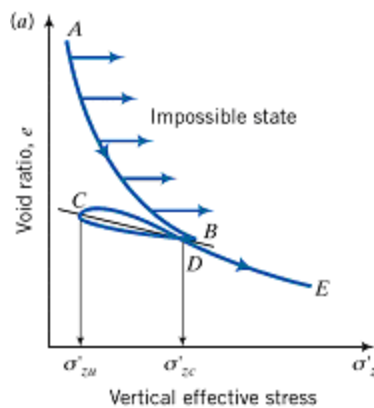
The *recompression index*, C_r , (slope BC in fig. 1b) is obtained from the plot of $\log \sigma'_z$ vs e as

$$C_r = \frac{e_2 - e_1}{\log \frac{(\sigma'_z)_2}{(\sigma'_z)_1}} = \frac{|\Delta e|}{\log \frac{(\sigma'_z)_2}{(\sigma'_z)_1}} \quad (\text{no units})$$

where the subscripts 1 and 2 denote two arbitrarily selected points on the URL.

The recompression index is appreciably smaller than the compression index. It is usually in the range of 1/10 to 1/5 of the compression index.

Normally consolidated, over-consolidated and under-consolidated clays



A **normally consolidated clay** is one which had not been subjected to a pressure greater than the present existing pressure. A soil is said to be over-consolidated if it had been subjected in the past to a pressure in excess of the present pressure.

The portion AB of the curve shown in figure represents the soil in normally consolidated condition.

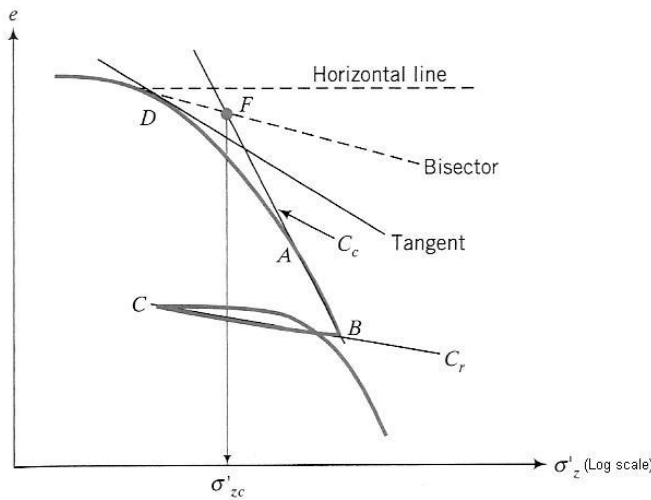
*The ratio of the past maximum vertical effective stress (σ'_{zm}) to the current vertical effective stress (σ'_{zo}) is called the **over consolidation ratio (OCR) or degree of overconsolidation.***

For example, if the past maximum vertical effective stress is 100 kPa and the current vertical effective stress is 100 kPa, the OCR = 1. A soil with OCR = 1 is called normally consolidated; a soil with $2 < OCR < 1$ is called a lightly over-consolidated soil; a soil with $OCR > 2$ is called an over-consolidated soil. A normally consolidated soil settles much more than an over-consolidated soil.

Under-consolidated clays: If a clay deposit has not reached equilibrium under the applied overburden loads, it is said to be under-consolidated. This normally occurs in areas of recent land fill.

Casagrande's method to determine the pre-consolidation effective stress

The maximum past effective stress to which the soil is subjected to in the past is called the pre-consolidation effective stress.

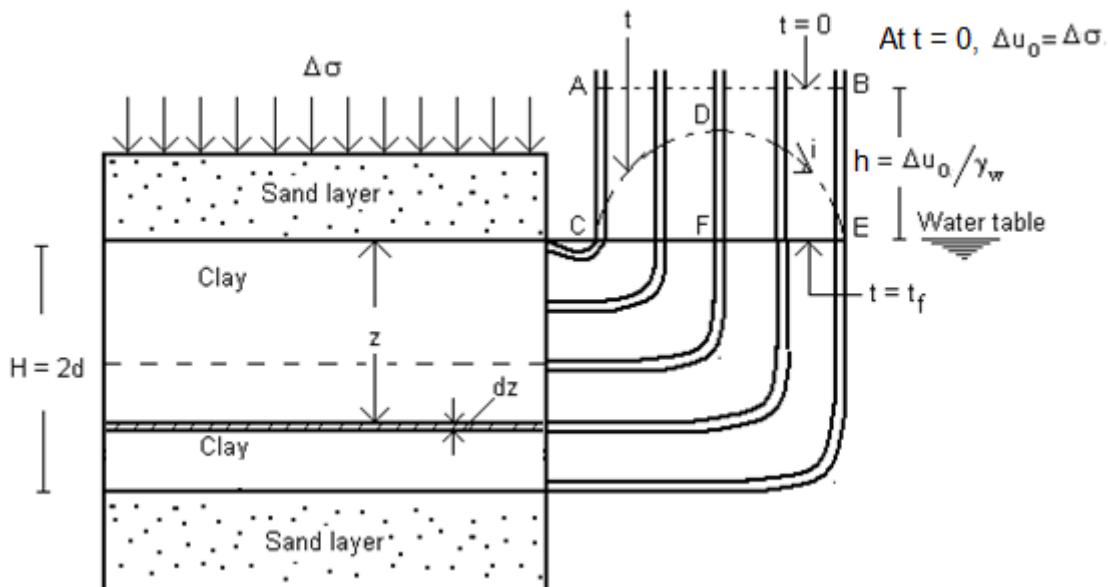


1. Identify the point of maximum curvature, D, on the initial part of the curve.
2. Draw a horizontal line through D.
3. Draw a tangent to the curve at D.
4. Bisect the angle formed by the tangent and horizontal line at D.
5. Extend backward the straight line portion of the curve (the normal consolidation line), BA, to intersect the bisector line at F.
6. The abscissa of F is the pre-consolidation stress, σ'_{zc} .

Terzaghi's one dimensional consolidation theory - Derivation of differential equation

Assumptions

1. The soil is homogeneous, isotropic and fully saturated
2. The solid particles and water in the voids are incompressible. The consolidation occurs due to expulsion of water from the voids.
3. The coefficient of permeability of the soil has the same value at all points, and it remains constant during the entire period of consolidation.
4. Darcy's law is valid throughout the consolidation process.



Consider a saturated clay layer of thickness $2d (= H)$ sandwiched between two layers of sand as shown in the figure below. When a uniform pressure $\Delta\sigma_z$ is applied on the surface of the top sand layer, the total stress developed at all points in the clay layer is increased by

$\Delta\sigma_z$. Initially the whole of the pressure is taken up by the water, and the excess pore water pressure of $\Delta\sigma_z/\gamma_w$ develops. Excess pore water pressure is shown on the right side of the figure. Assume that various points along the thickness of the clay layer are connected by flexible tubes to the piezometers. At $t = 0$, just after the application of the load, the excess pore water pressure Δu_0 is equal to $\Delta\sigma_z/\gamma_w$ throughout the layer. This is represented by the horizontal line AB. The excess pore water pressure is independent of the position of the water table.

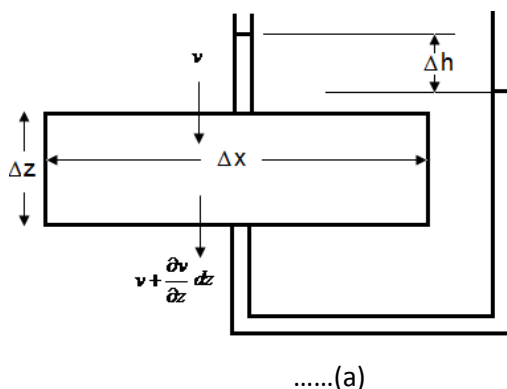
Water starts escaping towards the upper and lower sand layers due to excess pore water pressure developed. The excess pore water pressure at the top and bottom of the clay layer indicated by points C and E in the pressure diagram drops to zero. However, excess pore water pressure in the middle portion of the clay layer at D remains high. The curves indicating the distribution of excess pore water pressure are known as *isochrones*. The isochrone CDE indicates the distribution of excess pore water pressure at time t .

As the consolidation progresses, the excess pore water pressure in the middle of the clay layer decreases. Finally at time $t = t_f$, the whole of the excess pore water pressure has been dissipated, and the pressure distribution is indicated by the horizontal isochrone CFE.

Consider the equilibrium of an element of the clay at a depth z from its top at time t . The consolidation pressure $\Delta\sigma_z$ is partly carried by water and partly by solid particles as

$$\Delta\sigma_z = \Delta\sigma'_z + u$$

where $\Delta\sigma'_z$ is the pressure carried by solid particle and u is the excess pore water pressure.



The hydraulic gradient (i) at that depth is equal to the slope of the isochrone CDE at a horizontal distance z from the point C in the pressure diagram.

Thus,

$$i = \frac{\partial h}{\partial z} = \frac{\partial(u/\gamma_w)}{\partial z} = \frac{1}{\gamma_w} \left(\frac{\partial u}{\partial z} \right)$$

where u is the excess porewater pressure at depth z .

From Darcy's law, the velocity of flow at depth z is given by

$$v = ki = k \cdot \frac{1}{\gamma_w} \left(\frac{\partial u}{\partial z} \right)$$

The velocity of flow at the bottom of the element of thickness Δz can be written as

$$v + \frac{\partial v}{\partial z} \cdot dz = v + \frac{\partial}{\partial z} \left[\frac{k}{\gamma_w} \left(\frac{\partial u}{\partial z} \right) \right] dz$$

or
$$v + \frac{\partial v}{\partial z} \cdot dz = v + \frac{k}{\gamma_w} \left(\frac{\partial^2 u}{\partial z^2} \right) dz$$

Therefore,
$$\frac{\partial v}{\partial z} = \frac{k}{\gamma_w} \left(\frac{\partial^2 u}{\partial z^2} \right) \dots\dots\dots(1)$$

The discharge entering the element Q_{in} is

$$Q_{in} = v(\Delta x \times \Delta y)$$

Where Δx and Δy are the dimensions of the element in plan.

The discharge leaving the element Q_{out} is

$$Q_{out} = \left(v + \frac{\partial v}{\partial z} \cdot dz \right) (\Delta x \times \Delta y)$$

Therefore, the net discharge squeezed out of the element is given by

$$\Delta Q = \left[\left(v + \frac{\partial v}{\partial z} \cdot dz \right) - v \right] (\Delta x \times \Delta y)$$

or
$$\Delta Q = \frac{\partial v}{\partial z} (\Delta x \times \Delta y \times \Delta z) \dots\dots\dots(b)$$

As the water is squeezed out, the effective stress increases and the volume of the soil mass decreases. Therefore from the definition of modulus of volume compressibility,

$$\Delta V = -m_v V_o \Delta \sigma'_z$$

where V_o = initial volume of soil mass and $\Delta \sigma'_z$ = increase in effective stress.

The decrease in volume of soil mass per unit time is

$$\frac{\partial(\Delta V)}{\partial t} = -m_v (\Delta x \times \Delta y \times \Delta z) \frac{\partial(\Delta \sigma'_z)}{\partial t} \dots\dots\dots(c)$$

As the decrease in volume of soil mass per unit time is equal to the volume of water squeezed out per unit time, eqns. (b) and (c) give

$$\frac{\partial v}{\partial z} (\Delta x \times \Delta y \times \Delta z) = -m_v (\Delta x \times \Delta y \times \Delta z) \frac{\partial(\Delta \sigma'_z)}{\partial t}$$

or
$$\frac{\partial v}{\partial z} = -m_v \frac{\partial(\Delta \sigma'_z)}{\partial t} \dots\dots\dots(d)$$

Now,
$$\Delta \sigma'_z = \Delta \sigma_z - u$$

Or
$$\frac{\partial(\Delta\sigma'_z)}{\partial t} = \frac{\partial(\Delta\sigma_z)}{\partial t} - \frac{\partial(\Delta u)}{\partial t}$$

For a given pressure increment $\partial(\Delta\sigma_z) = 0$. Therefore, eqn.(d) becomes

$$\frac{\partial v}{\partial z} = -m_v \left(-\frac{\partial u}{\partial t} \right) = m_v \left(\frac{\partial u}{\partial t} \right) \dots\dots\dots(2)$$

From eqns. (1) and (2)

$$\frac{k}{\gamma_w} \left(\frac{\partial^2 u}{\partial z^2} \right) dz = m_v \left(\frac{\partial u}{\partial t} \right)$$

or
$$c_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \dots\dots\dots(3)$$

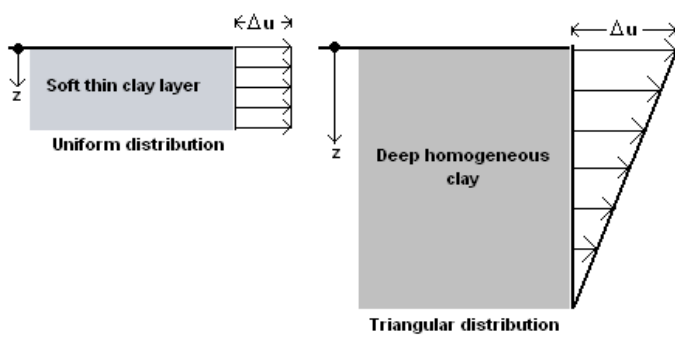
where c_v is the coefficient of consolidation and is given by

$$c_v = \frac{k}{m_v \gamma_w}$$

Eqn. (3) is the governing differential equation for one-dimensional consolidation. It allows to predict the changes in excess porewater pressure at various depths within the soil with time.

Solution of governing consolidation equation using Fourier series

The solution of any differential equation requires knowledge of the boundary conditions. By specifying the initial distribution of excess porewater pressures at the boundaries, solutions for the spatial variation of excess porewater pressures with time and depth can be obtained. Various distributions of porewater pressures within a soil layer are possible. Two of these are shown below. One of these is a uniform distribution of initial excess porewater pressure with depth. This may occur in a thin layer of fine-grained soils. The other is a triangular distribution. This may occur in a thick layer of fine-grained soils.



The boundary conditions for a uniform distribution of initial excess porewater pressure in which double drainage occurs are

When $t = 0, \Delta u = \Delta u_0 = \Delta\sigma_z$.

At the top boundary, $z = 0, \Delta u = 0$.

At the bottom boundary, $z = H = 2d$, where d is the length of the drainage path, $\Delta u = 0$

A solution for the governing one-dimensional consolidation equation satisfying these boundary conditions is obtained using the Fourier series,

$$u = \frac{4u_i}{\pi} \sum_{N=0}^{\infty} \frac{1}{(2N+1)} \left[\sin \frac{(2N+1)\pi z}{2d} \right] e^{-(2N+1)^2 \pi^2 T_v / 4} \dots\dots\dots(4)$$

where u_i is the initial excess pore water pressure and N is a positive integer with values from 0 to ∞ , and

$$T_v = \frac{c_v t}{d^2} \dots\dots\dots(5)$$

T_v is known as the time factor; it is a dimensionless term.

A plot of eqn.(4) gives the variation of excess pore water pressure with depth at different times.

Consider any arbitrarily selected isochrone at any time t or time factor T_v as shown in figure. At time $t = 0$ ($T_v = 0$), the initial excess porewater pressure, Δu_0 , is equal to the applied vertical stress throughout the soil layer. As soon as drainage occurs, the initial excess porewater pressure will immediately fall to zero at the permeable boundaries. The maximum excess porewater pressure will occur at the center of the soil layer because the drainage path there is the longest.

At time $t > 0$, the total applied vertical stress increment $\Delta \sigma_z$ at a depth z is equal to the sum of the vertical effective stress increment $\Delta \sigma'_z$ and the excess porewater pressure Δu_z . After considerable time ($t \rightarrow \infty$), the excess porewater pressure decreases to zero and the vertical effective stress increment becomes equal to the vertical total stress increment.

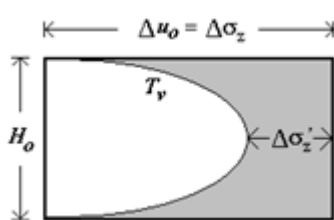
The degree of consolidation or consolidation ratio, u_z , which gives the amount of consolidation completed at a particular time and depth can mathematically expressed as

$$u_z = \frac{u_i - u}{u_i}$$

$$u_z = 1 - \sum_{N=0}^{\infty} \frac{2}{M} \sin \left(\frac{Mz}{d} \right) e^{-M^2 T_v}$$

Where $M = \frac{\pi}{2} (2N + 1)$

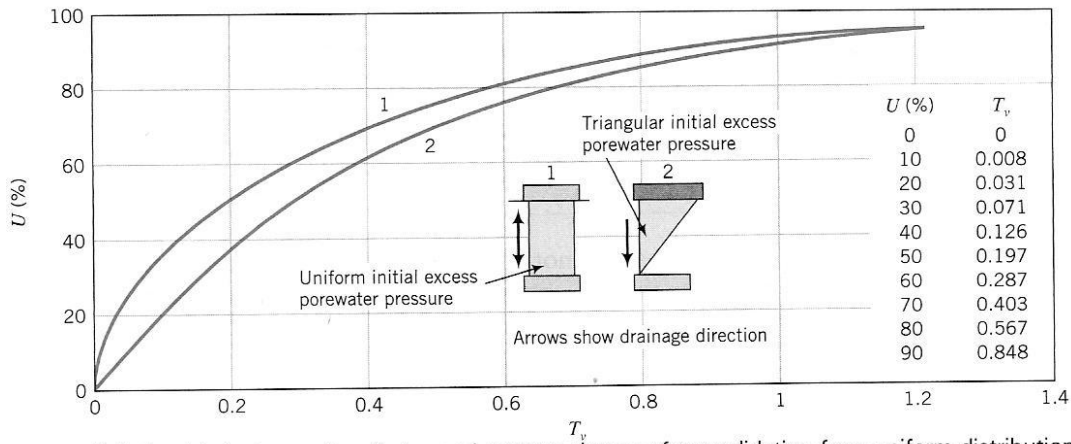
The consolidation ratio is zero everywhere at the beginning of the consolidation but increases to unity as the initial excess porewater pressure dissipates.



The average degree of consolidation, U , of a whole layer at a particular time rather than the consolidation at a particular depth is more often required. The shaded area in the figure represents the amount of consolidation of a soil layer at any given time. The average degree of consolidation can be mathematically expressed as

$$U = \frac{U_i - U_t}{U_i}$$

The figure below shows the variation of the average degree of consolidation with time factor T_v for a uniform and a triangular distribution of excess porewater pressure. A convenient set of equations for double drainage found by curve fitting this figure is



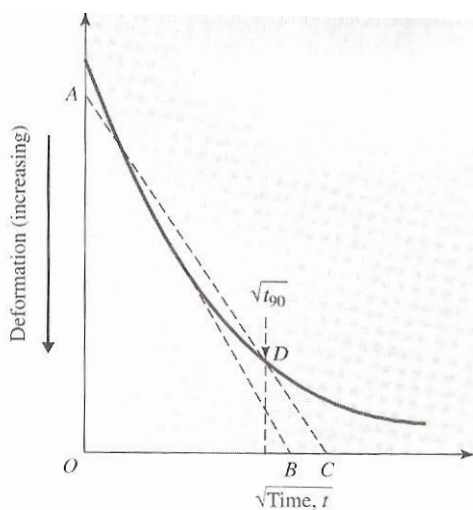
Relationship between time factor and average degree of consolidation for a uniform distribution and a triangular distribution of initial excess porewater pressure.

$$T_v = \frac{\pi}{4} \left(\frac{U}{100} \right)^2 \text{ for } U < 60\%$$

and $T_v = 1.781 - 0.933 \log(100 - U)$ for $U \geq 60\%$

The time factors corresponding to 50% and 90% consolidation are often used in interpreting consolidation test results. For 90% consolidation $T_v = 0.848$ and for 50% consolidation $T_v = 0.197$.

Determination of coefficient of consolidation



(a) Square root time method

1. Plot the dial gauge readings versus square root of times.
2. Draw a line AB through the early portion of the curve.
3. Draw a line AC such that $OC = 1.15 OB$. The abscissa of point D, which is the intersection of AC and the consolidation curve, gives the square root of time for 90% consolidation ($\sqrt{t_{90}}$).
4. For 90% consolidation, $T_v = 0.848$, therefore,

$$c_v = \frac{0.848d^2}{t_{90}}$$

where d is the length of the drainage path.

the soil before compression. If e_f is the void ratio of the sample after primary consolidation is complete, the decrease in void ratio is $(e_o - e_f)$. Then ΔH , the change in height of the consolidating layer or its settlement (ρ_{pc}) is given by

$$\frac{\Delta H}{H_o} = \frac{\Delta e}{1 + e_o}$$

$$\Delta H = \rho_{pc} = \frac{\Delta e}{1 + e_o} H_o$$

When the consolidation test results are plotted between void ratio and effective stress, the slope of the curve for the pertinent stress range, that is, the coefficient of compressibility a_v can be used in the settlement computation,

$$a_v = \frac{\Delta e}{\Delta \sigma'} \quad \text{or} \quad \Delta e = a_v \Delta \sigma'$$

$$\Delta H = \rho_{pc} = \left(\frac{a_v}{1 + e_o} \right) H_o \Delta \sigma'$$

But

$$m_v = \left(\frac{a_v}{1 + e_o} \right)$$

Therefore,

$$\rho_{pc} = m_v H_o \Delta \sigma'$$

Thus, the magnitude of settlement is a function of the soil compressibility m_v , boundary condition H_o (thickness) and the loading condition $\Delta \sigma'$.

The compression index C_c is

$$C_c = \frac{\Delta e}{\log \frac{\sigma'_f}{\sigma'_o}} \quad \text{or} \quad \Delta e = C_c \log \frac{\sigma'_f}{\sigma'_o}$$

$$\sigma'_f = \sigma'_o + \Delta \sigma'$$

where σ'_o is the present or initial effective stress or overburden pressure and $\sigma'_f = \sigma'_o + \Delta \sigma'$ additional stress $\Delta \sigma'$ induced by the imposed load.

The equation may

$$\Delta H = \rho_{pc} = C_c \frac{H_o}{1 + e_o} \log \frac{\sigma'_f}{\sigma'_o}$$

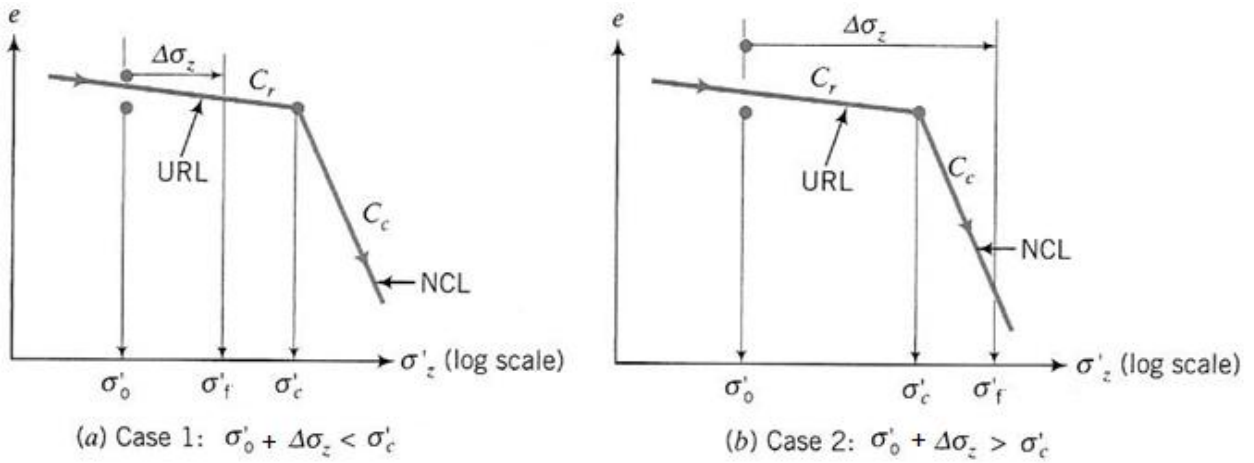
is the settlement equation for normally consolidated soils

C_c is constant independent of the stress range.

Primary consolidation of over-consolidated fine grained soils

If the soil is overconsolidated, two cases have to be considered depending on the magnitude

of $\Delta\sigma_z$. The curve in the $\log(\sigma'_z)$ versus e is approximated as two straight lines.



Case 1: The increase in $\Delta\sigma_z$ is such that $\sigma'_f = \sigma'_0 + \Delta\sigma_z$ is less than σ'_c (effective pre-consolidation stress). In this case consolidation occurs along the URL and

$$\rho_{pc} = \frac{H_o}{(1+e_o)} C_r \log\left(\frac{\sigma'_f}{\sigma'_0}\right); \sigma'_f < \sigma'_c$$

Case 2: The increase in $\Delta\sigma_z$ is such that $\sigma'_f = \sigma'_c + \Delta\sigma_z$ is greater than σ'_c (effective preconsolidation stress). In this case two components of settlement are considered – one along the URL and the other along the NCL. The primary consolidation settlement is

$$\rho_{pc} = \frac{H_o}{(1+e_o)} \left(C_r \log \frac{\sigma'_f}{\sigma'_0} + C_c \log \frac{\sigma'_f}{\sigma'_c} \right); \sigma'_f > \sigma'_c$$

or

$$\rho_{pc} = \frac{H_o}{(1+e_o)} \left(C_r \log(OCR) + C_c \log \frac{\sigma'_f}{\sigma'_c} \right); \sigma'_f > \sigma'_c$$

The primary consolidation settlement using m_v is $\rho_{pc} = H_o m_v \Delta\sigma_z$. Unlike C_c which is constant, m_v varies with stress levels. Therefore, an average value of m_v over the stress range σ'_c to σ'_f should be computed.

Procedure to calculate primary consolidation settlement

1. Calculate the current vertical effective stress (σ'_0) and the current void ratio (e_o) at

the center of the soil layer for which settlement is required.

2. Calculate the applied vertical stress increase ($\Delta\sigma_z$) at the centre of the soil layer using the appropriate method.
3. Calculate the final vertical effective stress, $\sigma'_f = \sigma'_c + \Delta\sigma_z$.
4. Calculate the primary consolidation settlement

- a. If the soil is normally consolidated ($OCR = 1$), the primary consolidation settlement is

$$\rho_{pc} = \frac{H_o}{(1 + e_o)} C_c \log\left(\frac{\sigma'_f}{\sigma'_c}\right)$$

- b. If the soil is over consolidated and $\sigma'_f < \sigma'_c$, the primary consolidation settlement is

$$\rho_{pc} = \frac{H_o}{(1 + e_o)} C_r \log\left(\frac{\sigma'_f}{\sigma'_o}\right)$$

- c. If the soil is over consolidated and $\sigma'_{fm} > \sigma'_{zc}$, the primary consolidation settlement is

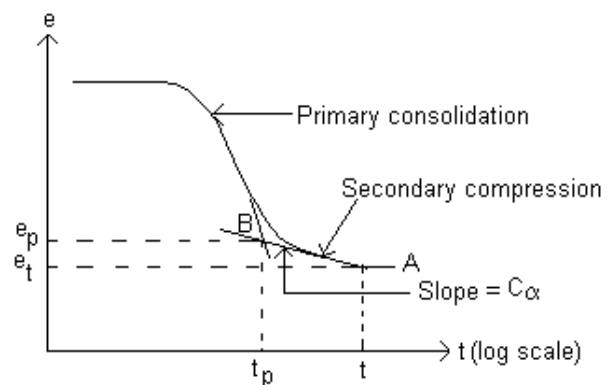
$$\rho_{pc} = \frac{H_o}{(1 + e_o)} \left(C_r \log(OCR) + C_c \log \frac{\sigma'_f}{\sigma'_c} \right)$$

where H_o is the thickness of the soil layer.

The primary consolidation settlement can also be calculate using m_v . However, unlike C_c , which is constant, m_v varies with the stress level. An average value of m_v should be calculated over the stress range σ'_{zo} to σ'_{fm} . The primary consolidation settlement, using m_v is $\rho_{pc} = H_o m_v \Delta\sigma_z$. The advantage of using this equation is that m_v is readily determined from the displacement data in consolidation tests; void ratio changes need not be calculated from the test data as required to determine C_c .

Secondary compression settlement

The physical reasons for secondary compression are not fully understood. One plausible explanation is the expulsion of water from micropores; another is viscous deformation of the soil structure. To determine the secondary compression index, C_α , a plot of void ratio versus logarithm of time is made from the consolidation test data as shown in the figure.



$$C_\alpha = -\frac{(e_t - e_p)}{\log(t/t_p)} = \frac{|\Delta e|}{\log(t/t_p)}; t > t_p.$$

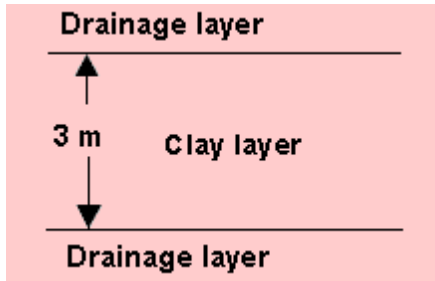
Where (t_p, e_p) is the coordinate at the intersection of the tangents to the primary consolidation and secondary compression parts of the void ratio versus logarithm of time

curve and (t, e_t) is the coordinate of any point on the secondary compression curve as shown in the figure above. The secondary consolidation settlement is

$$\rho_{sc} = \frac{H_o}{(1 + e_p)} C_\alpha \log\left(\frac{t}{t_p}\right)$$

Overconsolidated soils do not creep significantly but creep settlements in normally consolidated soils can be very significant.

Example 1: A 3 m thick layer of saturated clay in the field under a surcharge loading will achieve 90% consolidation in 75 days in double drainage conditions. Find the coefficient of consolidation of the clay.



Solution:

As the clay layer has two-way drainage, $H = 1.5 \text{ m} = 150 \text{ cm}$
 $t_{90} = 75 \text{ days} = 75 \times 24 \times 60 \times 60 \text{ seconds}$

For 90% consolidation ($U = 90\%$)

$$T_{90} = \frac{c_v \cdot t_{90}}{H^2} = 0.848$$

$$\therefore c_v = \frac{T_{90} \cdot H^2}{t_{90}}$$

$$= \frac{0.848 \times (150)^2}{75 \times 24 \times 60 \times 60}$$

$$= 2.94 \times 10^{-3} \text{ cm}^2 / \text{s}$$

Example 2: A 3 m thick clay layer in the field under a given surcharge will undergo 7 cm of total primary consolidation. If the first 4 cm of settlement takes 90 days, calculate the time required for the first 2 cm of settlement.

Solution:

Total consolidation = 7 cm

For 4 cm settlement, $U_1 = 4/7 \times 100 = 57.14\%$

For 2 cm settlement, $U_2 = 2/7 \times 100 = 28.57\%$

$t_1 = 90 \text{ days}$.

For,

$$U \leq 60\%$$

$$\frac{c_v t}{H^2} = T \propto U^2$$

$$\therefore \frac{t_1}{t_2} = \frac{U_1^2}{U_2^2}$$

$$\begin{aligned}\therefore t_2 &= \frac{U_2^2}{U_1^2} \times t_1 \\ &= \frac{(28.57)^2}{(57.14)^2} \times 90 \\ &= 22.5 \text{ days}\end{aligned}$$

Example 3: For a laboratory consolidation test on a soil specimen that is drained on both sides, the following were obtained:

Thickness of the clay specimen = 25 mm

$P_1 = 50 \text{ kN/m}^2$; $e_1 = 0.92$

$P_2 = 120 \text{ kN/m}^2$; $e_2 = 0.78$

Time for 50% consolidation = 2.5 min

Determine the soil permeability for the loading range.

Solution:

$$a_v = \frac{\Delta e}{\Delta \sigma'} = \frac{0.92 - 0.78}{120 - 50} = 0.002 \text{ m}^2 / \text{kN}$$

$$H = \frac{25 \text{ mm}}{2} = \frac{0.025 \text{ m}}{2} = 0.0125 \text{ m}$$

$$t_{50} = 2.5 \text{ min}$$

$$\begin{aligned}c_v &= \frac{T_{50} \cdot H^2}{t_{50}} \\ &= \frac{0.197 \times (0.0125)^2}{2.5} \\ &= 1.23 \times 10^{-5} \text{ m}^2 / \text{min}\end{aligned}$$

$$\begin{aligned}k &= \frac{c_v \cdot a_v \cdot \gamma_w}{1 + e_0} \\ &= \frac{1.23 \times 10^{-5} \times 0.002 \times 9.81}{1 + 0.92} \\ &= 1.26 \times 10^{-7} \text{ m / min}\end{aligned}$$

$$= \frac{0.197 \times (1.8788)^2}{(13)} \text{ cm}^2/\text{minute} = 0.053 \text{ cm}^2/\text{minute}$$

$$= 5.3 \times 10^{-2} \text{ cm}^2/\text{minute} = \frac{5.3}{60} \times 10^{-6} \text{ m}^2/\text{sec}$$

$$= 8.9 \times 10^{-8} \text{ m}^2/\text{sec. Ans.}$$

Note. The value of C_v obtained in the root-time method was also approximately the same.

Example 9.4. A consolidation test on a sample of clay having thickness of 2.3 cm indicates that half the ultimate compression occurs in the first 5 minutes. Under similar drainage conditions, how long will be required for a building on a 6 m layer of the same clay to experience half of its final settlement? Neglect secondary time effect.

Solution.

On Sample

$$t_1 = 5 \text{ minutes}$$

$$H_1 = 2.3 \text{ cm}$$

In Field

$$t_2 = ?$$

$$H_2 = 6 \text{ m} = 600 \text{ cm.}$$

Using Eq. (9.20), for the two similar soils to achieve the same degree of consolidation, we have

$$\frac{t_1}{t_2} = \left(\frac{H_1}{H_2} \right)^2$$

$$\frac{5}{t_2} = \left(\frac{2.3}{600} \right)^2$$

or

$$t_2 = 5 \times \left(\frac{600}{2.3} \right)^2 \text{ minutes} = \frac{5 \times (260.87)^2}{60 \times 24} \text{ days} = 236.29 \text{ days. Ans.}$$

Example 9.5. How many days would be required by a clay stratum 5 m thick, draining at both ends with an average value of coefficient of consolidation = $50 \times 10^{-4} \text{ cm}^2/\text{sec}$, to attain 50% of its ultimate settlement.

Solution. So long as the degree of consolidation U is less than 0.6, we can use the linear relationship between U and T_v , given by Eq. (9.33), as

$$T_v = \frac{\pi}{4} \cdot U^2$$

$$\therefore T_{v(50)} = \frac{\pi}{4} (0.5)^2 = 0.196.$$

Now, using Eqn. (9.32), the time taken to reach this 50% consolidation (t_{50}) is obtained as :

$$T_{v(50)} = \frac{C_v}{d^2} \cdot t_{50}$$

where d = maximum length of drainage path

$$= \frac{1}{2} \times 5 \text{ m} = 2.5 \text{ m} = 250 \text{ cm}$$

(\because it is draining at both ends)

$$0.196 = \frac{50 \times 10^{-4}}{(250)^2} \times t_{50}$$

or

$$t_{50} = \frac{0.196 \times 250 \times 250}{50 \times 10^{-4}} \text{ sec}$$

$$= \frac{0.196 \times 250 \times 250}{50 \times 60 \times 60 \times 24} \times 10^4 \text{ days} = 28.36 \text{ days. Ans.}$$

Example 9.6. A saturated soil has a compression index $C_c = 0.27$. Its void ratio at a stress of 125 kN/m^2 is 2.04, and its permeability is $3.5 \times 10^{-8} \text{ cm/sec}$. Compute : (a) the change in

void ratio if the stress is increased to 187.5 kN/m^2 ; (b) the settlement in (a) if the soil stratum is 5 m thick; and (c) Time required for 50% consolidation to occur if drainage is one way and time factor is 0.196 for 50% consolidation.

Solution. Using Eq. (9.12b), we have

$$e = e_0 - C_c \log_{10} \frac{p_o + \Delta p}{p_o}$$

where $p_o = 125 \text{ kN/m}^2$, $e_0 = 2.04$,

$$\Delta p = 187.5 - 125 = 62.5 \text{ kN/m}^2,$$

$$C_c = 0.27$$

$$\therefore e = 2.04 - 0.27 \left[\log_{10} \frac{187.5}{125} \right] = 1.993$$

$$\begin{aligned} \Delta e &= \text{change in void ratio} = e - e_0 \\ &= 1.993 - 2.04 = (-) 0.047 \end{aligned}$$

Hence, the void ratio is reduced by **0.047**. **Ans.**

(b) $\Delta H = ?$, $H_o = 5 \text{ m}$

Using Eq. (9.10), we have

$$\frac{\Delta H}{H_o} = \frac{\Delta e}{1 + e_0}$$

$$\therefore \frac{\Delta H}{500} = \frac{0.047}{1 + 2.04} = \frac{0.047}{3.04}$$

$$\therefore \Delta H = \frac{0.047}{3.04} \times 500 = 7.73 \text{ cm.}$$

Hence, settlement = **7.73 cm**. **Ans.**

(c) $T_{v(50)} = 0.196$, $t_{50} = ?$

$$d = H_o = 500 \text{ cm}$$

(as the drainage is one way, the length or drainage path equals full depth of soil).

Using Eqn. (9.32), we have

$$T_v = \frac{C_v}{d^2} t$$

or $0.196 = \frac{C_v}{(5 \text{ m})^2} \cdot t_{50}$... (i)

To determine t_{50} above, we should first determine C_v as follows :

$$\Delta H = m_v \cdot \Delta p \cdot H_o \quad \dots \text{Eq. (9.11)}$$

$$\therefore 7.73 \text{ cm} = m_v \times 62.5 \frac{\text{kN}}{\text{m}^2} \times (5 \times 100 \text{ cm})$$

or $m_v = \frac{7.73}{62.5 \times 500} \text{ m}^2/\text{kN} = 2.47 \times 10^{-4} \text{ m}^2/\text{kN}$

Further using Eq. (9.30), $K = C_v \cdot m_v \cdot \gamma_w$ we have

$$\therefore 3.5 \times 10^{-10} \frac{\text{m}}{\text{sec}} = C_v \times (2.47 \times 10^{-4}) \frac{\text{m}^2}{\text{kN}} \times 9.81 \frac{\text{kN}}{\text{m}^3} \quad (\because K = 3.5 \times 10^{-8} \text{ cm/s ; given})$$

or $C_v = \frac{3.5 \times 10^{-10}}{2.47 \times 10^{-4} \times 9.81} \text{ m}^2/\text{sec} = 1.44 \times 10^{-7} \text{ m}^2/\text{sec}$

Substituting this value of C_v in (i) above, we get

$$0.196 = \frac{1.44 \times 10^{-7} \text{ m}^2/\text{sec}}{(5 \text{ m})^2} \times t_{50}$$

or
$$t_{50} = \frac{0.196 \times 25}{1.44 \times 10^{-7}} \text{ sec} = 3.403 \times 10^7 \text{ sec}$$

$$= \frac{3.403 \times 10^7}{60 \times 60 \times 24} \text{ days} = 393.84 \text{ days. Ans.}$$

Example 9.6. Find the time required for 50% consolidation in a soil stratum, 9 m thick with a pervious strata on top and bottom. Also determine the coefficient of consolidation given that $K = 10^{-9} \text{ m/sec}$; $e_0 = 1.5$, $\alpha_v = 0.003 \text{ m}^2/\text{kN}$, Time factor = 0.2.

Solution. $d = \frac{H}{2} = \frac{9 \text{ m}}{2}$; as it is draining at both ends = 4.5 m

$t_{50} = ?$, $C_v = ?$, $T_v = 0.2$, $K = 10^{-9} \text{ m/sec}$.
 $e_0 = 1.5$, $\alpha_v = 3 \times 10^{-3} \text{ m}^2/\text{kN}$

Using Eq. (9.32), we have $T_v = \frac{C_v}{d^2} t$

or $t = \frac{T_v}{C_v} \cdot d^2$

$\therefore t_{50} = \frac{0.2}{C_v} (4.5 \text{ m})^2 \quad \dots(i)$

To determine C_v , we have

$K = C_v \cdot m_v \cdot \gamma_w$ (Eq. 9.30)

where $m_v = \frac{\alpha_v}{1 + e_0}$ (from Eq. 9.2)

$= 3 \times 10^{-3} \frac{\text{m}^2}{\text{kN}} \left[\frac{1}{1 + 1.5} \right] = 1.2 \times 10^{-3} \text{ m}^2/\text{kN}$

$\therefore 10^{-9} \text{ m/sec} = C_v \times (1.2 \times 10^{-3} \frac{\text{m}^2}{\text{kN}}) \times 9.81 \frac{\text{kN}}{\text{m}^3}$

Hence, $C_v = \frac{10^{-9}}{1.2 \times 10^{-3} \times 9.81} \text{ m}^2/\text{sec} = 8.49 \times 10^{-8} \text{ m}^2/\text{sec}$

Now from (i), $t_{50} = \frac{0.2}{C_v} (4.5 \text{ m})^2$
 $= \frac{0.2}{8.49 \times 10^{-8}} \frac{\text{sec}^2}{\text{m}^2} \times (4.5)^2 \text{ m}^2$
 $= \frac{0.2 \times 4.5^2 \times 10^8}{8.49} \text{ secs} = 552 \text{ days. Ans.}$

Example 9.7. A soil of specific gravity 2.65 has a moisture content of 18% when fully saturated. 1.9 cm thick sample of this soil tested in a consolidometer shows a compression of 0.050 cm when the load is increased from 40 kN/sq m to 80 kN/sq m. Compute the compression index of the soil.

Solution. $G = 2.65$, $w = 0.18$
 $S = 1$ (fully saturated)

$\therefore e = \frac{w \cdot G}{S} = 0.18 \times 2.65$

$\therefore e = 0.477$.

Now, compression $\Delta H = 0.050 \text{ cm}$.

$H_0 = \text{Thickness of sample} = 1.9 \text{ cm}$.

Now, using $\frac{\Delta H}{H_0} = \frac{\Delta e}{1 + e_0}$, we get

$$\frac{0.050}{1.9} = \frac{\Delta e}{1 + 0.477}$$

$$\therefore \Delta e = \frac{1.477 \times 0.050}{1.9} = 0.0389$$

Now, use Eq. : $\Delta e = C_c \log_{10} \frac{p_0 + \Delta p}{p_0}$,

$$\text{where } p_0 = 40 \text{ kN/m}^2$$

$$\Delta p = 80 - 40 = 40 \text{ kN/m}^2$$

$$\therefore 0.0389 = C_c \log_{10} \frac{40 + 40}{40} = C_c \log_{10} 2$$

$$\text{or } C_c = \frac{0.0389}{\log_{10} 2} = 0.129$$

Hence,

$$C_c = 0.129. \text{ Ans.}$$

Example 9.8. The loading period for a building extended from July 1987 to July 1989. In July 1992, the average measured settlement was found to be 113 mm. It is known that the ultimate settlement will be about 360 mm. Estimate the settlement in July 1997. Assume double drainage to occur.

Solution. When loading is applied over a certain period of time, then all time calculations are done by assuming the middle of the construction period as datum. Thus, in this problem, the building was constructed between July 1987 to July 1989. Hence, the datum would be July 1988. All further timings would be calculated from this datum.

Hence, 113 mm settlement observed in July 1992 would be after a period of 4 years from this datum (July 1988).

$$\text{Settlement } S_1 \text{ after } t_1 \text{ (4 yrs)} = 113 \text{ mm} = 0.113 \text{ m}$$

$$\text{Total settlement (S i.e. } \Delta H) = 360 \text{ mm} = 0.360 \text{ m}$$

$$\text{Settlement in July 1997, i.e. after 9 yrs from datum} = S_2 \text{ at } t_2 \text{ (9 yrs)} = ?$$

$$\text{Now, } U_1 = \text{degree of settlement after } t_1 \text{ (4 yrs) time} = \frac{0.113 \text{ m}}{0.360 \text{ m}} = 0.314$$

$$U_2 = \text{degree of settlement after } t_2 \text{ (9 yrs) time} = \frac{S_2}{0.360} = 2.778 S_2$$

Now, from Eq. (9.33),

$$T_v = \frac{\pi}{4} U^2 \text{ (for } U \leq 0.60)$$

$$\therefore T_{v1} = \frac{\pi}{4} (0.314)^2 \quad \dots(i)$$

$$\text{And } T_{v2} = \frac{\pi}{4} (2.778 S_2)^2 \quad \dots(ii)$$

(assuming U_2 to be ≤ 0.6 ; to be checked after computing S_2)

$$\text{Also } T_v = \frac{C_v}{d^2} \cdot t$$

$$\therefore T_{v1} = \frac{C_v}{d^2} \cdot t_1, \quad T_{v2} = \frac{C_v}{d^2} \cdot t_2$$

$$\therefore \frac{T_{v1}}{T_{v2}} = \frac{t_1}{t_2}$$

$$\text{or } \frac{\frac{\pi}{4} (0.314)^2}{\frac{\pi}{4} (2.778 S_2)^2} = \frac{4 \text{ yrs}}{9 \text{ yrs}}$$

$$\text{or } \frac{(0.314)}{2.778 S_2} = \sqrt{\frac{4}{9}} = \frac{2}{3}$$

$$\therefore S_2 = \frac{3}{2} \times \frac{0.314}{2.778} = 0.1696 \text{ m}$$

Let us check back that $U_2 \leq 0.6$

$$\text{So } U_2 = 2.778 S_2 = 2.778 \times 0.1696 = 0.471$$

(which is less than 0.6, and hence our assumption at (A) is OK)

Hence, the expected settlement after 9 yrs, i.e. in July 1997
= 0.1696 m = 169.6 mm. Ans.

Example 9.9. A 3 m thick clay layer beneath a building is overlain by a permeable stratum and is underlain by an impervious rock. The coefficient of consolidation of the clay was found to be $0.025 \text{ cm}^2/\text{min}$. The final expected settlement for the layer is 8 cm.

- (i) How much time will it take for 80% of total settlement to take place?
- (ii) Determine the time required for a settlement of 2.5 cm.
- (iii) What will be the settlement in 6 months?

u	T_u
0	0
0.1	0.008
0.2	0.031
0.3	0.071
0.4	0.126
0.5	0.196
0.6	0.287
0.7	0.403
0.8	0.567
0.9	0.848
1.0	∞

(Engineering Services, 1994)

Solution. 100% settlement = $\Delta H = 8 \text{ cm}$

80% settlement = $80\% \times 8 \text{ cm} = 6.4 \text{ cm}$

$$t_{80\%} = ?$$

Use the general Eqn. (9.32) as :

$$T_v = \frac{C_v}{d^2} \cdot t,$$

where T_v for 80% settlement or consolidation = 0.567

[\because From the given values of U and T_v , for $U = 0.8$ (80%)]

d = max. length of drainage path

= full depth of clay, because it will drain only on one side, since impervious rock exists below the clay layer, and pervious soil exists above it

= 3 m

$C_v = 0.025 \text{ cm}^2/\text{min}$ (given)